

"On the Distribution of Stress and Strain in the Cross-section of a Beam." By JOHN MORROW, M.Sc. (Vict.), Lecturer in Engineering, University College, Bristol. Communicated by Professor HELE-SHAW, F.R.S. Received October 27,—Read November 26, 1903.

Introduction.

Our knowledge of the strains produced in materials by different kinds and combinations of stress rests mainly on theoretical considerations. Much accurate experimental work has been done in the observation of direct tensile strains, but little attention has been given to the lateral strain accompanying a simple tensile or compressive strain, or to the lateral strain occurring in a bar under bending forces. The latter, indeed, has, perhaps, never before been measured in metal specimens.

For some time past the writer has been making experiments with the object of showing that instruments can be constructed capable of measuring these lateral strains with considerable accuracy. The subject is of great interest to the elastician, as it not only provides a method of determining elastic coefficients, but shows the degree of applicability of mathematical results; and, further, in view of the well-known discrepancies which exist between experiment and theory, the subject is also of no small practical importance.

The work here described is confined mainly to experiments on iron beams, and has had for its objects the following:—

1. The design of a comparatively simple instrument which can be used for the accurate measurement of the lateral displacements in the section of a beam;
2. The determination of the amount and distribution of this strain in iron beams; and
3. The determination of Poisson's ratio from the observed lateral displacements.

Description of the Apparatus.

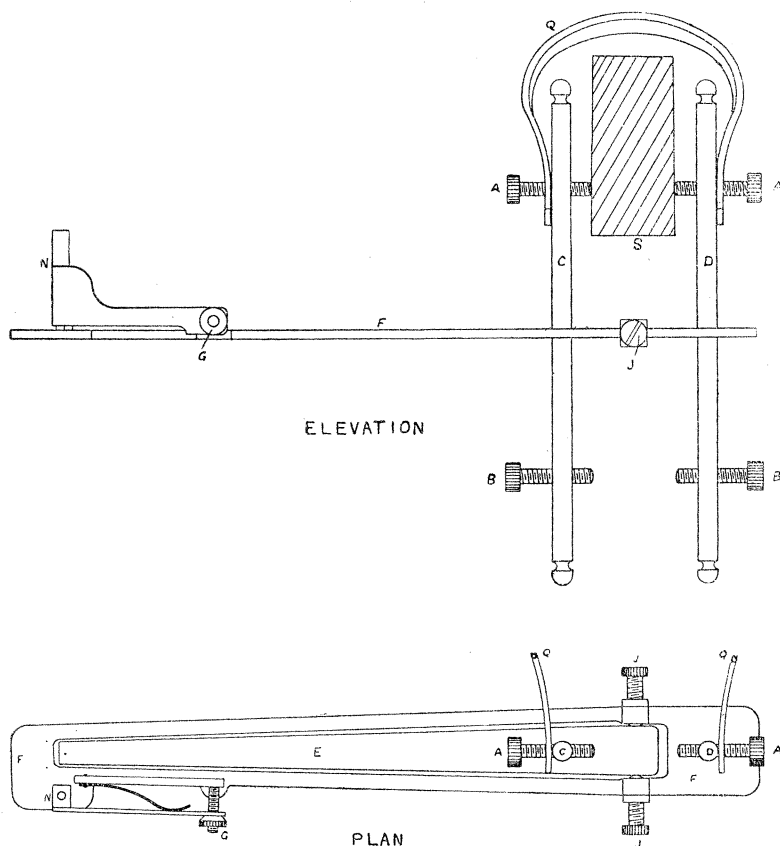
Whilst making some experiments on the lateral constriction of tie-bars,* it occurred to the author that, with a modified form of the apparatus then used, measurements of the deformation of the section of a beam might be made.

After trials of several different forms of instrument, and various ways of applying the load, it was decided that a long bar should be placed in the testing machine, and the readings taken with the mirror apparatus shown in fig. 1. The specimen S is in section

* 'Phil. Mag.,' Sixth Series, vol. 6, No. 34, p. 417.

in the elevation. It is gripped by the two set-screws AA, each of which is screwed through one of the vertical rods C and D. These rods pass respectively through the pieces E and F, and are rigidly attached to them. CE and DF thus form a pair of levers pivoted together at JJ. Any motion of AA, therefore, will be transmitted to the extreme ends of E and F. Thus if the specimen contract, AA

FIG. 1.



approach one another, and E will rise relatively to F; the relative motion of E and F will be a measure of the contraction of the specimen.

To obtain the amount of this relative motion, two mirrors, M and N, are used. They are made of specially prepared optically plane glass. The former is supported on three hard steel needle points, two of which rest on F, and the third on E, as shown in the plan. The other

mirror, N, is attached to F on a vertical spindle, about which it can be turned by the fine adjustment screw G.

When the instrument is set up, a scale is placed in an upright position in front of the mirrors, and some distance away, and the two images of the scale are brought together in the field of a telescope by means of the screw G. A convenient mark on the N image is then taken as an index, and the reading coinciding with it on the other image is noted each time the load on the beam is altered. From the difference between the readings before and after applying a load, the transverse strain is calculated across the breadth of the beam.

The instrument may be placed either below the specimen, when the screws AA are used, or above, when BB perform exactly the same functions.

The pressure between the screws and the specimen is maintained by a stiff spring, Q and the weight of the instrument taken up by attaching a thin cord to C, which, passing over a light pulley, carries a balance weight. It will be seen that the instrument is free from strains, which vary with the load on the specimen.

The dimension from the axis to the plane of the tilting mirror was 20 cm., while that between the centre-line of the set-screws and the axis was 5 cm. A magnification of four was thus obtained in the instrument itself. For a long time consistent reading could not be obtained, and it was only after very many trials of the instrument and alterations in its design that the causes of the inconsistency were successively eliminated.

The apparatus was very sensitive to vibrations, and it was soon found that accurate readings could only be obtained when the machinery in the adjacent laboratories was at rest. Vibrations caused by traffic some distance away were quite noticeable.

The most troublesome factor was probably the small unavoidable amount of jerk accompanying the application of the load. Occasionally this would cause a very slight displacement of the tilting mirror, which could not be detected on looking in the telescope, but nevertheless was sufficient to vitiate the accuracy of the readings. Under these circumstances it was impossible ever to depend on a single observation. The load was invariably applied and removed several times, and if the readings were consistent, and returned each time to their original value when the load was taken off, they were accepted as correct.

Any imperfection in the adjustment of the instrument or in the manner of applying the load was always made evident by the erratic nature of the readings.

Method of Making the Experiments.

For the first series of experiments, a long cast-iron bar, 2·833 cm. broad and 6·452 cm. deep, was placed in the "Wicksteed" testing machine, in the Engineering Laboratory at University College, Bristol. It was supported on knife edges at each end of a span of 91·44 cm. (36 inches), and was loaded in the centre in the ordinary way.

At a distance of from about 7·5 to 9·0 cm. from the centre of the span lines were carefully scribed on the sides of the beam, marking the places where the set-screws of the strain-measuring instrument were to rest. One of these lines was at the middle of the depth, and the others at distances of 0·635, 1·905, and 3·175 cm., both above and below the mid-point.

The instrument was attached just clear of the ends of these lines, and 36·80 cm. from one of the planes of supports. Thus the section examined was at a distance from the point of application of the load exceeding the depth of the beam, and it was hoped that this would be sufficient to secure immunity from the effects of local strains due to surface loading. Under these circumstances, each ton applied at the centre would produce a bending couple of 18,694 kilogram-centimetres at the section under observation.

The effective distance between the needle points of the tilting mirror, as given by a reading microscope, was 0·6160 cm., and the normal distance of the scale from the mirrors 202·7 cm.

The total magnification was, therefore, 2632 for the first experiments, but it was, of course, re-determined whenever the position of the scale was altered.

The scale was divided to $\frac{1}{40}$ th of an inch, and, with the telescope used, $\frac{1}{16}$ th of these divisions could be readily estimated. The observed numbers are, therefore, given to the nearest $\frac{1}{444560}$ of a centimetre.

When taking a series of readings for a certain change of load, the instrument was first applied with the points AA at the lowest mark, and a number of readings taken each time the load was applied or removed. The set-screws were then moved up to the next mark, and the readings continued. When the centre was reached, the instrument was taken off and re-applied above the specimen, with the screws BB at the top marks, and observations were continued down to the centre again.

The log sheet was kept as follows:—

Table I.—Loads 1 to $1\frac{1}{2}$ tons. Cast-iron Beam No. 1.

Load.	Scale reading.	Diffs.	Mean diff.
Tons.			
1·0	110·0		
1·5	103·9	6·1	
1·0	110·1	6·2	
1·5	104·0	6·1	6·15
1·0	110·2	6·2	
1·5	104·0	6·2	
1·0	110·1	6·1	

Position of instrument, 2nd line from top. Index mark, 50. Right-hand scale descends when load is increased.

Results of the Experiments.

The experiments for each load were carried out in the manner described above, and the load was at first advanced by $\frac{1}{2}$ -ton intervals. Some difficulty was experienced, however, in obtaining satisfactory results for the first $\frac{1}{2}$ ton, and ultimately the zero readings were rejected and a series taken between $\frac{1}{4}$ and $\frac{1}{2}$ ton. This is summarised in Table II.

Table II.—Load $\frac{1}{4}$ to $\frac{1}{2}$ ton.

Number of mark.	Distance from centre.	Mean scale differences.	Lateral strain in specimen.
1	3·175	4·68	$3·92 \times 10^{-5}$
2	1·905	3·24	2·71
3	0·635	1·20	1·00
4	0·00	0·00	0·00
5	− 0·635	− 1·10	− 0·92
6	− 1·905	− 2·95	− 2·47
7	− 3·175	− 4·87	− 4·08

The minus sign denotes that the lateral strain was a contraction.

The values, of course, refer to the elastic state of the bar only, permanent set having been removed as far as possible before the readings were commenced, by alternate application and removal of the load.

The experiments were then continued in an exactly similar manner, the loads being increased by $\frac{1}{2}$ tons until 2 tons was reached, and the last set of readings being taken between 2 and $2\frac{1}{4}$ tons.

Fracture took place at 2·46 tons. It showed sound material and occurred about 1 cm. from the middle of the span.

Table III contains the results of these experiments. At the centre two sets of readings were obtained at each load. These, in general, differed but slightly, and the mean values are given. The slight differences were probably due to the limits of accuracy having been reached.

Table III.

Position of instrument.	Increments of strain at different loads. (Scale readings.)				
	$\frac{1}{4}$ to $\frac{1}{2}$ ton.	$\frac{1}{2}$ to 1 ton.	1 to $1\frac{1}{2}$ tons.	$1\frac{1}{2}$ to 2 tons.	2 to $2\frac{1}{4}$ tons.
1	4·68	9·63	10·12	10·30	5·73
2	3·24	5·82	6·15	6·17	3·07
3	1·20	1·92	2·09	2·19	0·84
4	0·00	0·06	0·03	-0·12	-0·31
5	-1·10	-1·88	-2·13	-2·29	-1·69
6	-2·95	-5·25	-5·60	-6·00	-3·45
7	-4·87	-9·40	-9·00	-8·55	-3·96

The column headed "Position of the Instrument" refers to the number of the mark on the side of the beam in line with which the instrument was attached.

A minus sign denotes that the lateral strain was a contraction.

Distribution of Strain.

At the lower loads the maximum lateral strain per unit of load added is on the tensile side of the beam, slightly greater than that on the compressive side. As the load increases, however, the maximum lateral strain per unit of added load in the tensile fibres decreases, whereas that in the compressive fibres constantly increases. Hence, at fracture, this strain is very much greater on the compressive side than on the tensile.

The curves in fig. 2 are plotted from the second and sixth columns of Table III, and show the increment of lateral strain at different points in the depth of the beam. The full line refers to loads of $\frac{1}{4}$ and $\frac{1}{2}$ ton, and the broken line 2 to $2\frac{1}{4}$ tons. For the other loads the curves are given in fig. 3. They would be intermediate between the two curves of fig. 2.

Considering the strain curves for the tensile side of the beam given in figs. 2 and 3, it will be seen that at the lower loads a straight line is obtained, and that as the load is increased the lines become more and more curved in the direction of decreasing strain in the outer layers. This curvature is first noticed in Numbers 3 and 4, but not till the last curve, Number 5, is it very remarkable. On the compression side at the

FIG. 2.

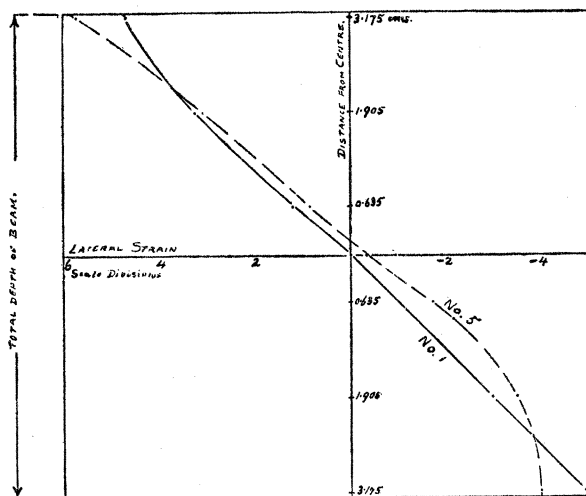
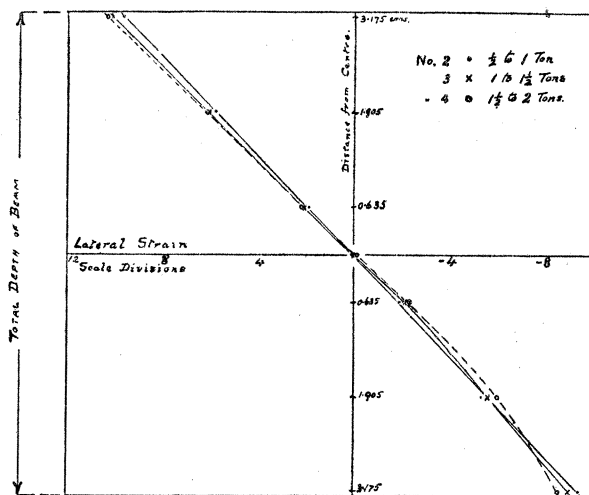


FIG. 3.



lowest load a similar curvature exists (see fig. 2), and although all the readings were carefully checked, the result was practically the same.* In Numbers 2, 3, and 4, however, the line is straight, whilst in Number 5 there is a pronounced curvature in the opposite direction.

* It has been suggested that this unexpected curvature might be accounted for by additional strains due to the comparative smallness of the distance from the loaded part.

The general conclusions to be drawn from these curves are, firstly, that as the load increases, the increment of lateral strain in the outer tensile layers becomes less, and that this is accompanied by a much increased increment in the less stressed fibres nearer the neutral surface; and, second, that on the compression side, with higher loads, the increment of lateral strain in the outer fibres increases.

To what extent these actions take place before fracture can only be inferred from the experimental results. The greatest changes occurred between 2 and $2\frac{1}{4}$ tons, that is with a bending moment at the section under observation of not more than four-fifths of that which the beam was capable of resisting before fracture at the middle of the span; and between these limits the effects noticed would become much more remarkable.

The most striking fact brought out is, as will be seen later, that the values of the strains thus obtained are considerably lower than would be expected from theoretical considerations.

This fact appeared so important that it was decided to further verify the result by the examination of a second cast-iron beam.

Experiments on Cast-iron Beam No. 2.

The beam used for this further investigation was 7·578 cm. deep and 3·212 cm. broad. The span was 91·44 cm. as before, and the section examined 37 cm. from one of the supports. Readings were taken at the middle points of the depth and 1·2, 2·4, and 3·6 cm. both above and below these points.

The load was varied between $\frac{1}{2}$ and 1 ton only. In all other respects the procedure of the previous experiments was repeated.

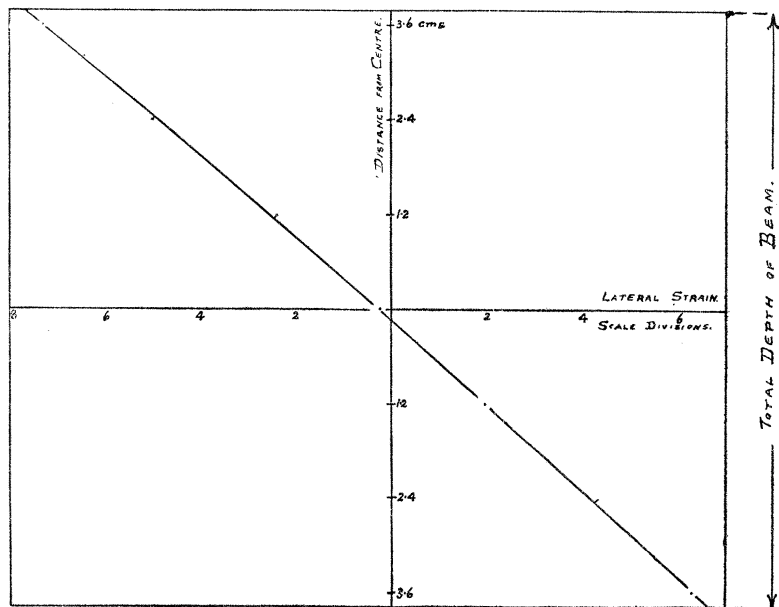
Table IV contains the results in differences of scale readings, the first column showing the distance of the point under observation from the middle of the depth. The curve is given in fig. 4.

It will be shown that these results corroborate those obtained from the beams previously described.

Table IV.

Position of instrument.	Differences of scale readings.
3·6	+ 7·30
2·4	+ 5·00
1·2	+ 2·40
0·0	+ 0·25
1·2	- 1·97
2·4	- 4·34
3·6	- 6·28

FIG. 4.

*Experiments on a Wrought-iron Beam.*

The following experiments were made on a wrought-iron bar 3.122 cm. broad and 7.614 cm. deep. The span was the same as before, namely 91.44 cm., and the section under observation was 37 cm. from one of the supports.

In this case the readings were taken at the middle of the depth and at distances above and below of 1.2, 2.4, and 3.4 cm.

The method of procedure was exactly the same as that with the cast-iron beams.

Table V.

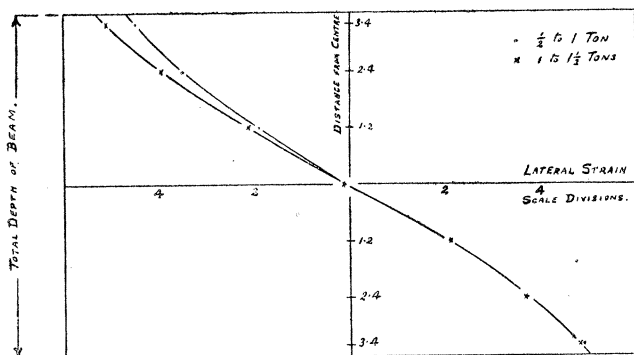
Distance from centre in cms.	Difference of scale readings.		Increments of strain per $\frac{1}{2}$ ton.	
	$\frac{1}{2}$ to 1 ton.	1 to $1\frac{1}{2}$ tons.	$\frac{1}{2}$ to 1 ton.	1 to $1\frac{1}{2}$ tons.
3.4	+4.50	+5.10	$+3.30 \times 10^{-5}$	$+3.74 \times 10^{-5}$
2.4	+3.52	+3.96	+2.58	+2.91
1.2	+1.90	+2.12	+1.39	+1.56
0.0	+0.10	+0.12	+0.07	+0.09
1.2	-2.08	-2.12	-1.53	-1.56
2.4	-3.73	-3.68	-2.74	-2.70
3.4	-4.87	-4.80	-3.57	-3.52

The experiments were carried out with two increments of load, namely from $\frac{1}{2}$ —1 ton and 1—1 $\frac{1}{2}$ tons.

Table V contains the results both in differences of scale readings and in increments of strain per half ton.

The curves are plotted in fig. 5, and it will be seen that, whilst on the tension side the two curves practically agree, on the compression side the increment of strain is greater for the higher load.

FIG. 5.



Relations between Stress and Lateral Strain.

In order to appreciate the results of the foregoing experiments, and to compare them with those which would be expected from purely theoretical considerations, it became necessary to investigate the relations which obtain between the lateral strain and the stress producing it, in specimens under direct tensile and compressive forces.

To obtain these relations for the material of the first cast-iron beam specimens were cut from the less strained portions of the beam itself.

The tensile piece was 1.956 cm. in diameter, and was of uniform cross-section for a length of about 25.0 cm. The ends were screwed, and gripped in the testing machine by the ordinary ball and socket arrangements.

The instrument used was very similar in principle to that employed for measuring the lateral strain in the beams. It has been fully explained and illustrated in a paper published in the 'Philosophical Magazine' (6th series, vol. vi, p. 417, October, 1903).

The total magnification was 2744.5, and the load was advanced $\frac{1}{2}$ ton at a time.

The lateral strain per $\frac{1}{2}$ ton of load was then calculated by dividing the difference of scale readings by 84540, and the corresponding stress was 169.07 kilogrammes per square centimetre.

The results are tabulated and reduced in Table VI (Appendix).

The method of applying the load was similar to that described in the paper quoted above, so that the lateral strains do not include that due to permanent set. The specimen was not tested to fracture.

It will be seen that the elastic lateral strain is proportional to the stress. The ratio may be expressed

Lateral strain per kilogramme per square centimetre = $20\cdot66 \times 10^{-8}$.

The compression piece was cylindrical in shape, 2·436 cm. in diameter, and about 7·5 cm. high. The measurement of change of diameter was made about the middle of the specimen, in order that the effects of any suppression of the dilatation at the ends might be avoided.

The load was applied exactly as in the case of the tensile piece. Table VII (Appendix) contains the results. The total magnification was 3002.

The ratio of stress to strain is not quite independent of the stress. Near the origin the lateral strain per kilogramme per square centimetre = $20\cdot91 \times 10^{-8}$.

In conjunction with my colleague, Mr. E. L. Watkin, M.A., I have made a large number of experiments on these stress-strain relations for cast-iron. We hope to publish an account of them shortly. Some of the specimens used were cast at the same time and from the same ladle as the second beam described in this paper.

In Table VIII (Appendix) are embodied the results obtained from two of these, the particular ones chosen giving average and fairly representative values. The tension piece was 1·958 cm. in diameter, and the compression specimen 2·499 cm. diameter and 7·5 cm. high.

For comparison with the wrought-iron beam the relations were obtained from separate specimens of similar material.

In the case of tension the lateral strain has been found to be directly proportional to the stress when within the elastic limit, and the experiments described in a previous paper* gave, for the material in question, the value $13\cdot80 \times 10^{-8}$ for the lateral strain per kilogramme per square centimetre.

The wrought-iron compression specimen was similar to those of cast iron, 2·404 cm. diameter and 7·5 cm. high. The load was applied by $\frac{1}{2}$ ton intervals up to 2 tons, and then by tons to 4 tons. The lateral dilatations were not quite proportional to the stresses applied. The results are given in Table IX (Appendix).

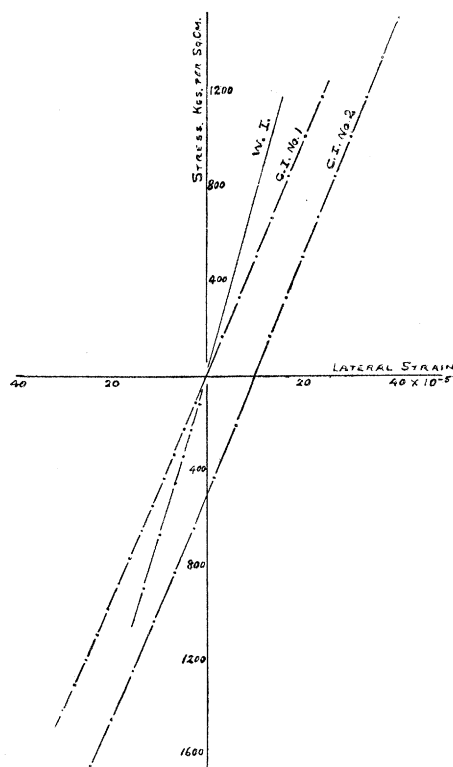
The relations between stress and lateral strain for all three cases are shown graphically in the curves of fig. 6. The origin of the curve for comparison with Cast-iron Beam No. 2 is displaced to the right a distance of 10×10^{-5} unit.

* See 'Phil. Mag.,' Sixth Series, vol. 6, No. 34, p. 417.

Distribution of Stress in Beams.

Assuming, as in the Coulomb-St. Venant theory, that the relations which hold between direct stress and lateral strain in tensile and compressive tests are, at the lower loads, the same as those which obtain between the stresses parallel to the axis of a beam and the corresponding strains at right-angles to that direction, the distribution of stress over the cross-section of a beam may be inferred from the observed lateral strains.

FIG. 6.



Thus, Table III gives the differences of scale readings measuring the lateral displacements in the first cast-iron beam. In Table X we have the total scale differences due to the different loads. This has been obtained directly from Table III by addition; and for the zero readings it has been supposed that from $0-\frac{1}{4}$ ton the displacements would follow the same law as those between $\frac{1}{4}$ and $\frac{1}{2}$ ton. The actual lateral strains are then calculated as in Table XI. They are given

by dividing the numbers in Table X by the magnification and the breadth of the beam, and multiplying the result by the scale unit.

By comparing these strains with Tables VI and VII (Appendix), or with the curve in fig. 6, we can find the direct stresses which must have accompanied them. These are given in Table XII for loads up to $1\frac{1}{2}$ tons.

Proceeding in a similar way for Cast-iron Beam No. 2 and for the wrought-iron beam, we obtain the second column of Table XIII and the last two columns of Table V for the actual strains per $\frac{1}{2}$ ton, and comparing these with fig. 6, we get the stresses given in the Appendix in the third column of Table XIII for the cast-iron beam, and in Table XIV for the wrought-iron beam.

The degree in which these stresses agree with those obtained by the ordinary theory of the bending prism, as given by Coulomb and generalised by Saint Venant, may be taken as a measure of the applicability of that theory. Thus Table XIV contains the theoretical values of the stresses calculated for the wrought-iron beam, and it will be seen that the agreement with the experimental results is not very good at the higher stresses.

The theoretical stresses for the cast-iron beams are given in Tables XV and XIII (Appendix).

Discussion of the Results for Cast Iron.

The theoretical stresses are not calculated for loads higher than $1\frac{1}{2}$ tons, as above that point some of the values obtained would be greater than the actual tensile strength of the material.

At the lower loads the lateral strains, and the stresses inferred from them, are generally lower than those obtained theoretically.

The amount of this discrepancy can best be seen by comparing Tables XII and XV.

It is also noteworthy that as the load is increased the results show a distinct shifting of the position of the neutral axis from the tensile towards the compressive side of the beam.

For loads over $1\frac{1}{2}$ tons not only is the theoretically calculated maximum stress greater than the ultimate stress the material will stand before fracture, but the values obtained by measuring the strains are also considerably greater than the maximum lateral strain which occurs in a tensile specimen.

The strain at fracture of a tensile specimen is about 35×10^{-5} . This is nearly equalled by the lateral strain noted at $2\frac{1}{4}$ tons load on the beam, whilst the maximum lateral strain at fracture would be greatly in excess of this value.

Saint Venant* assumes that, since all materials are capable of a small

* See Saint Venant's 'Navier' (Paris, 1864), p. 178.

amount of flow when nearing the point of rupture, the curve of distribution of stress over the cross-section of the beam becomes parallel to the plane of the section when rupture commences.

This suggests that the higher lateral strains may be due to the amount of plasticity of the material just before fracture.

In a tensile test no such plasticity can be observed. This, however, does not invalidate the assumption, as it would be necessary to check the plastic elongation, in order to measure it before complete rupture occurred.

One effect of the permanent set in the direct strains may be to produce a more or less profound change in the distribution of stress over the section. This might give rise to the existence of lateral strains, which, though perhaps elastic in their nature, would not be included in the observed values of the experiments.

It has been pointed out* that, when the load on an overstrained beam is reduced, initial stresses may result. The strain in the outer layer might then be reduced by an amount greater than the real elastic strain due to the load removed.

It appears, however, to be far more probable that the departures from theory which have been noticed are due to the inapplicability of the theory employed; that is, mainly to the facts that (1) the Saint Venant solution implies different conditions of loading and end-fixing,† and (2) it is based on an assumption of the absolute proportionality of stress to strain.

In this connection it should be remarked that a different distribution of stress and strain over the section would not necessarily alter the deflection or curvature of the centre line of a beam due to a given bending couple.‡

The author hopes to deal more fully with these and other questions in a future paper.

Note on the Determination of Poisson's Ratio from Bending Experiments.

The instrument described in this paper provides a method not hitherto available for the determination of the values of the "stretch-squeeze" ratio for different materials.

The method appears at present only suitable for those materials and stresses to which the ordinary theory of bending may be applied.

It depends on the relation between the lateral strain at any section, and the deflection of the centre line produced by the applied bending couple.

Consider a beam, of uniform rectangular section, supported at the

* See 'Encycl. Brit.,' vol. 22, article "Strength of Materials."

† See paper by L. N. G. Filon, 'Phil. Trans.,' A, vol. 201, p. 83.

‡ See Wiedemann's 'Annalen der Physik und Chemie,' vol. 52, 1894, paper by W. Voigt, p. 536.

ends, and subjected to bending by applying a transverse force at the centre of its span. Let the origin be at one end and let

- x be taken in the direction of the length,
- y = the distance from the neutral surface,
- P = the stress at (x, y) ,
- W = the load applied at the centre,
- δ = the deflection of the centre line at x ,
- E = Young's modulus for the material,
- σ = Poisson's ratio,
- l, b, d = the length, breadth, and depth of the beam.

If at any section, $x \geq l/2$, the change produced in the breadth of the beam be measured when the load is applied, and, at the same time, the deflection of the centre line of the beam at that point be observed, then the angle turned through by the side of the beam

$$= \tan^{-1} \frac{\sigma P b}{2 E y},$$

and the deflection, δ , is given by

$$\delta = \frac{W l^2}{16 E I} x - \frac{W}{12 E I} x^3.$$

Combining these we obtain an expression from which σ can be at once determined, namely, angle turned through by side of beam

$$= \tan^{-1} \frac{12 \sigma \delta b}{3 l^2 - 4 x^2}.$$

The left-hand side of this equation is obtained from the mean straight line through the points representing the lateral strain in the beam.

In conclusion the author must express his thanks to Messrs. F. C. Prentice and C. M. Rushton for assistance rendered at various times during the preparation of this paper.

APPENDIX.

Table VI (see fig. 6).—Tension and Lateral Contraction of Test Piece cut from Cast-iron Beam No. 1.

Load.	Lateral strain per $\frac{1}{2}$ ton (scale readings).	Total lateral strain (scale divisions).	Lateral strain (elastic only).	Stress, kilos. per sq. cm.
Tons.				
$\frac{1}{2}$	2·95	2·95	$3·49 \times 10^{-5}$	169
1	2·95	5·90	6·98	338
$1\frac{1}{2}$	2·94	8·84	10·46	507
2	2·95	11·79	13·95	676
$2\frac{1}{2}$	2·97	14·76	17·46	845
3	2·96	17·72	20·96	1014
$3\frac{1}{2}$	2·95	20·67	24·45	1183

Table VII (see fig. 6).—Lateral Dilatation of Compression Specimen cut from Cast-iron Beam No. 1.

Load.	Increments of strain (scale divisions).	Total lateral strain (scale divisions).	Lateral strain (elastic only).	Stress, kilos. per sq. cm.
Tons.				
$\frac{1}{2}$	2·60	2·60	$2·26 \times 10^{-5}$	109
1	2·56	5·16	4·48	218
$1\frac{1}{2}$	2·63	7·79	6·76	327
2	2·63	10·42	9·05	436
$2\frac{1}{2}$	2·60	13·02	11·31	545
3	2·73	15·75	13·68	654
$3\frac{1}{2}$	2·75	18·50	16·06	763
4	2·67	21·17	18·38	872
$4\frac{1}{2}$	2·70	23·87	20·73	981
5	2·73	26·60	23·10	1090
$5\frac{1}{2}$	2·70	29·30	25·44	1199
6	2·75	32·05	27·83	1308
$6\frac{1}{2}$	2·80	34·85	30·26	1417

Table VIII (see fig. 6).—Stress and Lateral Strain in Cast Iron for comparison with Beam No. 2.

Tension.		Compression.	
Stress kilos. per sq. cm.	Lateral strain (elastic only).	Stress kilos. per sq. cm.	Lateral strain (elastic only).
139	$3 \cdot 37 \times 10^{-5}$	207	$3 \cdot 95 \times 10^{-5}$
337	6·80	414	8·12
506	10·25	621	12·48
675	13·65	828	16·83
843	17·08	1035	21·28
1012	20·45	1243	25·70
1181	23·85	1450	30·14
1350	27·32	1657	34·53
1518	30·75	1864	38·98
1687	34·25	2071	43·45
		2278	47·85
		2485	52·30

Table IX (see fig. 6).—Stress and Lateral Strain in Wrought Iron under Compression.

Load.	Increments of strain (scale divisions).	Total strain, scale divisions (elastic only).	Elastic lateral strain.	Stress, kilos. per sq. cm.
Tons.				
$\frac{1}{2}$	1·80	1·80	$1 \cdot 58 \times 10^{-5}$	112
1	1·84	3·64	3·20	224
$1\frac{1}{2}$	1·85	5·49	4·83	336
2	1·90	7·39	6·50	448
3	3·80	11·19	9·85	672
4	3·82	15·01	13·21	895

Table X.—Cast-iron Beam No. 1.

Point in depth of beam.	Total strains at different loads. (Scale divisions.)				
	0 to $\frac{1}{2}$ ton.	1 ton.	$1\frac{1}{2}$ tons.	2 tons.	$2\frac{1}{2}$ tons.
1	9·36	18·99	29·11	39·41	45·14
2	6·48	12·30	18·45	24·62	27·69
3	2·40	4·32	6·41	8·60	9·44
4	0·00	0·06	0·09	— 0·03	— 0·34
5	— 2·20	— 4·08	— 6·21	— 8·50	— 10·19
6	— 5·90	— 11·15	— 16·75	— 22·75	— 26·20
7	— 9·74	— 19·14	— 28·14	— 36·69	— 40·65

Table XI.—Cast-iron Beam No. 1.

Point in depth of beam.	Total strains at different loads.				
	$\frac{1}{2}$ ton.	1 ton.	$1\frac{1}{2}$ tons.	2 tons.	$2\frac{1}{2}$ tons.
1	7.83×10^{-5}	15.89×10^{-5}	24.36×10^{-5}	32.98×10^{-5}	37.78×10^{-5}
2	5.42	10.29	15.44	20.60	23.17
3	2.01	3.62	5.36	7.20	7.90
4	0.00	0.05	0.08	— 0.03	— 0.28
5	— 1.84	— 3.41	— 5.20	— 7.11	— 8.53
6	— 4.94	— 9.33	— 14.02	— 19.04	— 21.93
7	— 8.15	— 16.02	— 23.55	— 30.70	— 34.02

Table XII (Table XI converted to Stresses).

Point in depth of beam.	Stresses in kilos. per sq. cm.		
	$\frac{1}{2}$ ton.	1 ton.	$1\frac{1}{2}$ tons.
1	— 375	— 760	— 1152
2	— 259	— 492	— 738
3	— 96	— 173	— 257
4	0	— 2	— 3
5	90	165	252
6	239	452	679
7	395	775	1140

— sign indicates compressive stress.

Table XIII.—Comparison of Theoretical Distribution of Stress with that obtained from the Lateral Strains in Cast-iron Beam No. 2.

Distance from centre in cm.	Stresses corresponding to observed lateral strains.		Calculated stress, kilos. per sq. cm.
	Lateral strain.	Stress.	
3.6	5.21×10^{-5}	— 270	— 290
2.4	3.57	— 187	— 194
1.2	1.71	— 90	— 97
0.0	0.18	— 9	00
1.2	— 1.41	69	97
2.4	— 3.10	153	194
3.6	— 4.48	221	290

Table XIV.—Comparison of Theoretical Distribution of Stress with that obtained from the Lateral Strains in the Wrought-iron Beam.

Distance from centre in cm.	Stresses corresponding to observed lateral strains.		Calculated stress, kilos. per sq. cm.
	$\frac{1}{2}$ to 1 ton.	1 to $1\frac{1}{2}$ tons.	
3·4	— 231	— 253	— 278
2·4	— 181	— 199	— 196
1·2	— 99	— 107	— 98
0·0	— 5	— 6	0
1·2	111	113	98
2·4	198	196	196
3·4	259	255	278

Table XV.—Theoretically-calculated Stresses in Cast-iron Beam No. 1.

Distance from centre in cm.	Calculated stresses in kilos. per sq. cm.		
	$\frac{1}{2}$ ton.	1 ton.	$1\frac{1}{2}$ tons.
3·175	— 460	— 920	— 1380
1·905	— 276	— 552	— 828
0·635	— 92	— 184	— 276
0·00	00	00	00
0·635	92	184	276
1·905	276	552	828
3·175	460	920	1380